

# A Log Volume Formula for Exporters

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## Summary

This paper shows that the volume estimate of export logs would be more accurate using a taper value in the 3D formula than volumes derived from the presently used Japanese Agricultural Standard.

The 3D formula with a taper term based on small-end diameter and length was derived and tested on a data set of 9163 radiata pine logs measured between 2003 and 2010. Tests showed no error trends on log length or diameter, and the total volume was estimated to be within -0.1% of the 3D volume. Two further data sets, collected between 1946 and 1980, were used to test the formula on sectionally measured logs from older age classes and un-thinned stands. Overall the formula showed a small volume bias of +1.1% on 35281 radiata pine logs, and a bias of +4.3% on 5733 Douglas fir logs. Although some error trends were evident, the volume estimates were satisfactory over the range of export log dimensions.

It is shown that the new (2D) formula shows less volume bias and smaller error trends on diameter and length than the JAS and Chinese National formulae.

## Introduction

In the year ended March 2011, 11.7 million cubic metres of logs were exported from New Zealand softwood plantations. Almost all of the logs were individually measured using the Japanese Agricultural Standard (JAS) (Anon, 1967) the method used in New Zealand since the 1980's.



Figure 1. Logs being loaded for the Chinese market

The procedures for scaling for JAS in New Zealand are defined in (Ellis and Elliott 2001). JAS understates the cubic volume of short logs and those of small diameter by as much as 40% of the actual volume, while longer, larger Douglas fir logs with little taper maybe over estimated by 20% (Ellis and Elliott 2001, Ellis et al. 1996). Mensuration and scaling systems in New Zealand provide estimates of the under-bark cubic content of logs and trees (Ellis 2005, Goulding 2005). As a result, forest owners require conversion from JAS volume to true cubic for stand volume reconciliation.

All logs for export from New Zealand are measured on truck and only the small-end diameter and length are used for the calculation of volume. JAS is calculated by assuming a rectangular solid shape with square rather than circular ends and for logs greater than 6m, with a pre-determined standard allowance for the increase in log diameter from small-end to large-end. Thus actual log taper has a large impact on the accuracy of the volume estimate from small-end diameter and length.

China is now the largest market for New Zealand export logs. There is a Chinese National Standard (Anon, 1984) for the calculation of log volumes, with differing formulae for length and diameter combinations.. These calculate volume assuming circular rather than square cross-sections, but retain predetermined assumptions on log taper and hence differ from JAS and the true cubic volume. The complete JAS and Chinese volume formulae are shown in Appendix 1.

When export logs arrive at their destination they are often rescaled by another method such as Hiragoku or Guo Biao. Conversion tables are used to convert NZ JAS to the local scale.

The 3-D formula (Ellis 1982) was designed to replace the large number of site specific, small-end diameter and length tables in use at that time. The 3-D formula estimates inside bark volume from measurements of both end diameters and log length. "The 3-D method is unique in that the formula adjusts stem form based on taper, and thus allows the scaler to measure the butt-cut end of a log with normal flare" (Fonseca, 2005). The 3-D formula also allowed the measurement of large-end diameter to be replaced by an estimated average log taper. Samples of logs were used to provide a taper estimate for various

Table 1. Basic data

	Length (metres)	Diameter in centimetres		Taper (cm/m)
		Small-end	Large-end	
Minimum	2.22	8.00	12.0	0.13
Mean	6.40	32.6	40.6	1.30
Maximum	12.32	74.5	95.5	5.43

regions, stands and age classes. With the large scale of export volumes from various regions and forests in the form of multiple grade types, it is not practical to apply sampled taper factors. It would be practical to use this application for export logs as long as a standard estimate of taper could be applied without the need for regular updates for all grades of log.

This study investigates the application of a small-end diameter and length based taper factor and the resulting volume estimates.

**Basic data**

Logs that were measured between 2003 and 2010 were compiled from throughout New Zealand to form a data set. The length of each log, and under-bark end diameters were measured to the nearest centimetre. The end measurement of diameter was the average of the shortest under-bark diameter through the geometric centre and the diameter at right angles to this. Taper is the difference between the small-end and large-end diameter per unit length and expressed in cm/m. The grade of log is broadly representative of the popular export grades of which “A” and “K” grade contributes about 50% and “pulp” 15% of the export volume. Table 1 shows the range of dimensions for the 9163 logs.

**Development of a taper function**

The average taper of 1.3 cm/m on the basic data was placed in the 3D formula to examine the volume estimates over the small-end diameter and length range. Volume estimates showed error trends on both small-end diameter and length. There was only a weak correlation between taper and small-end diameter in the data, but as it wasn’t desirable to use a fixed taper allowance, it was necessary to estimate taper from small-end diameter (and length).

Various combinations of diameter and length were examined, and stepwise regression employed to predict log taper that provided the least error trends when used in the 3D formula. The final taper formula (standard error = 0.518) is shown in Formula 1.

$$T = 2.112 + 0.00071D^2 - 0.0405D - 0.047L \text{ Formula 1}$$

where D= average small-end diameter in centimetres  
 L = length in metres  
 T = log taper in centimetres per metre

Formula 1 was then substituted for the taper term in the 3D formula. In this paper this formula with the taper term will be referred to as the 2D formula, shown in Appendix 1.

**Results**

The 2D formula was tested on various length, diameter and taper combinations on the basic data. Volume errors on length are shown in Figure 2 along with errors using fixed taper, the JAS formula (Formulae 3 and 4), and the Chinese National Standard (Formulae 5-8).

It can be seen in Figure 2 that the percentage volume differences between 3D and 2D volume are around zero for most of the length range. Both JAS and Chinese formula show error trends on length. Figure 3 shows the comparative volume errors on small-end diameter

In Figure 3 the percentage differences between 3D and 2D volumes are small over the diameter range. The 2D formula with fixed taper, the Chinese and JAS formulae all show tendencies to underestimate the volume of small diameter logs.

The 2D formula was also tested against two historical data sets comprised of logs from unthinned stands of trees in older age classes. Actual log volumes in those data sets were derived from sectional measurements and Smalian’s formula. The under-bark diameters for each section were measured with a girth tape. Formula errors on length for 2D, fixed taper, JAS, and Chinese formula are shown in Figure 4. The 2D formula shows consistent volume over most of the length range, with differences within 2% of actual. All of the other methods show error trends on length.

Figure 5 shows volume errors on small-end diameter.

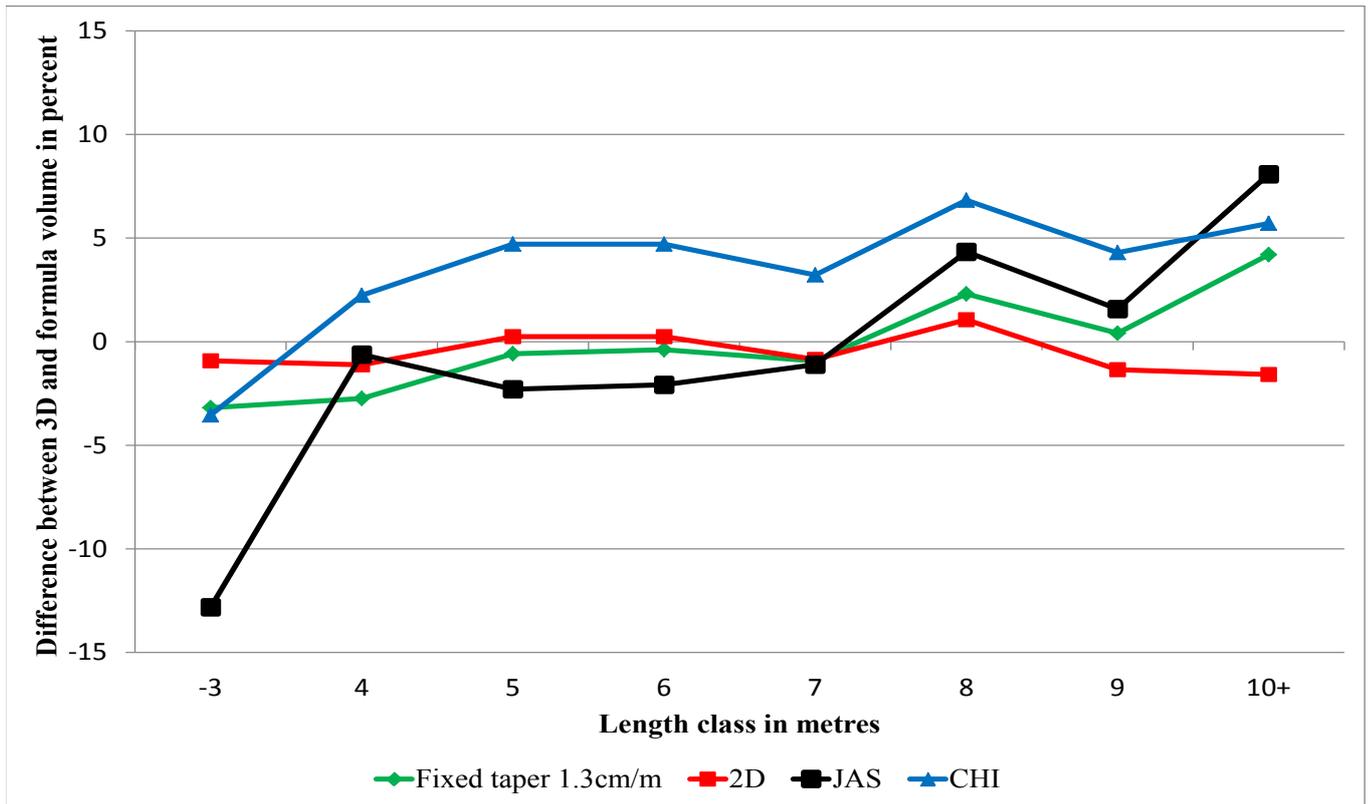


Figure 2. Volume errors on length class

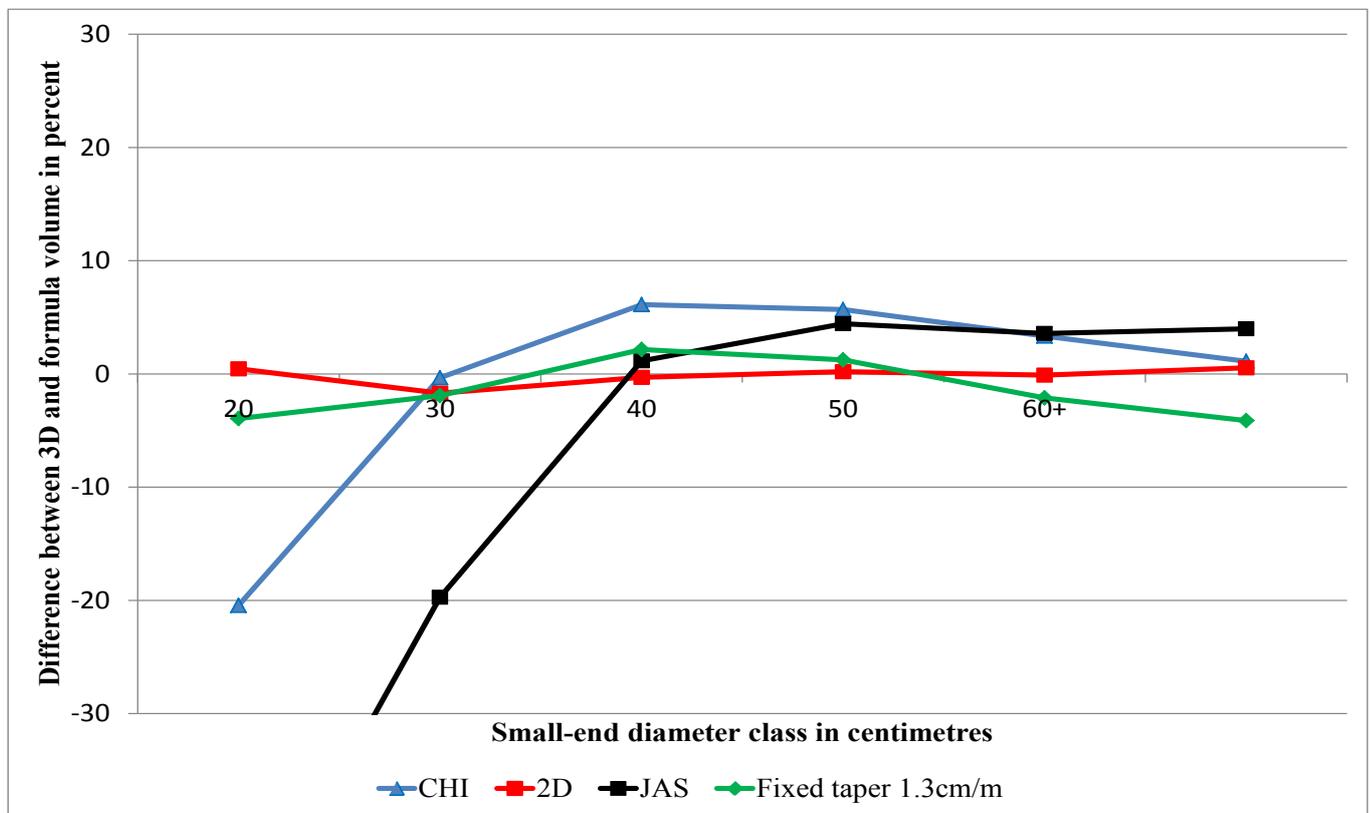


Figure 3. Volume errors on small-end diameter.

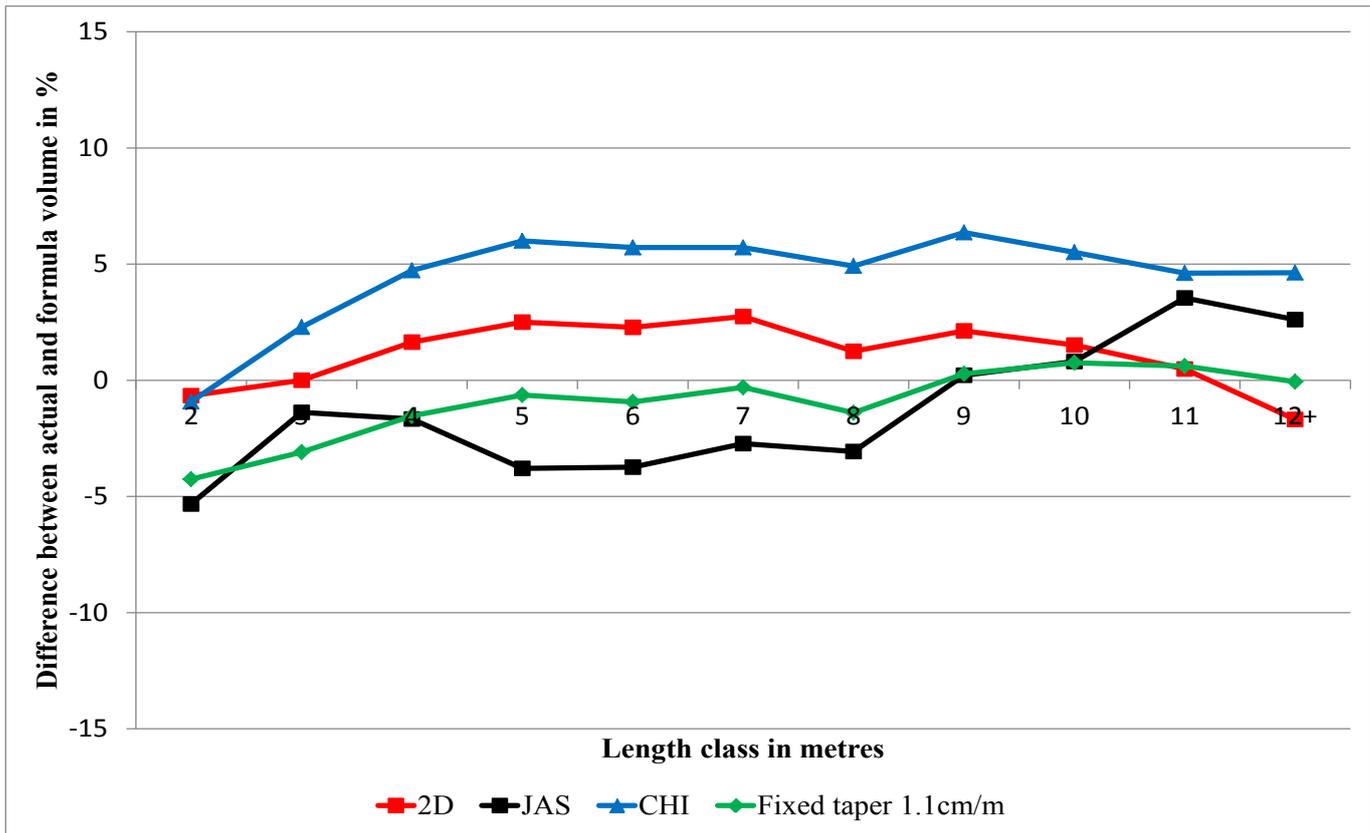


Figure 4. Volume errors on length for external data set

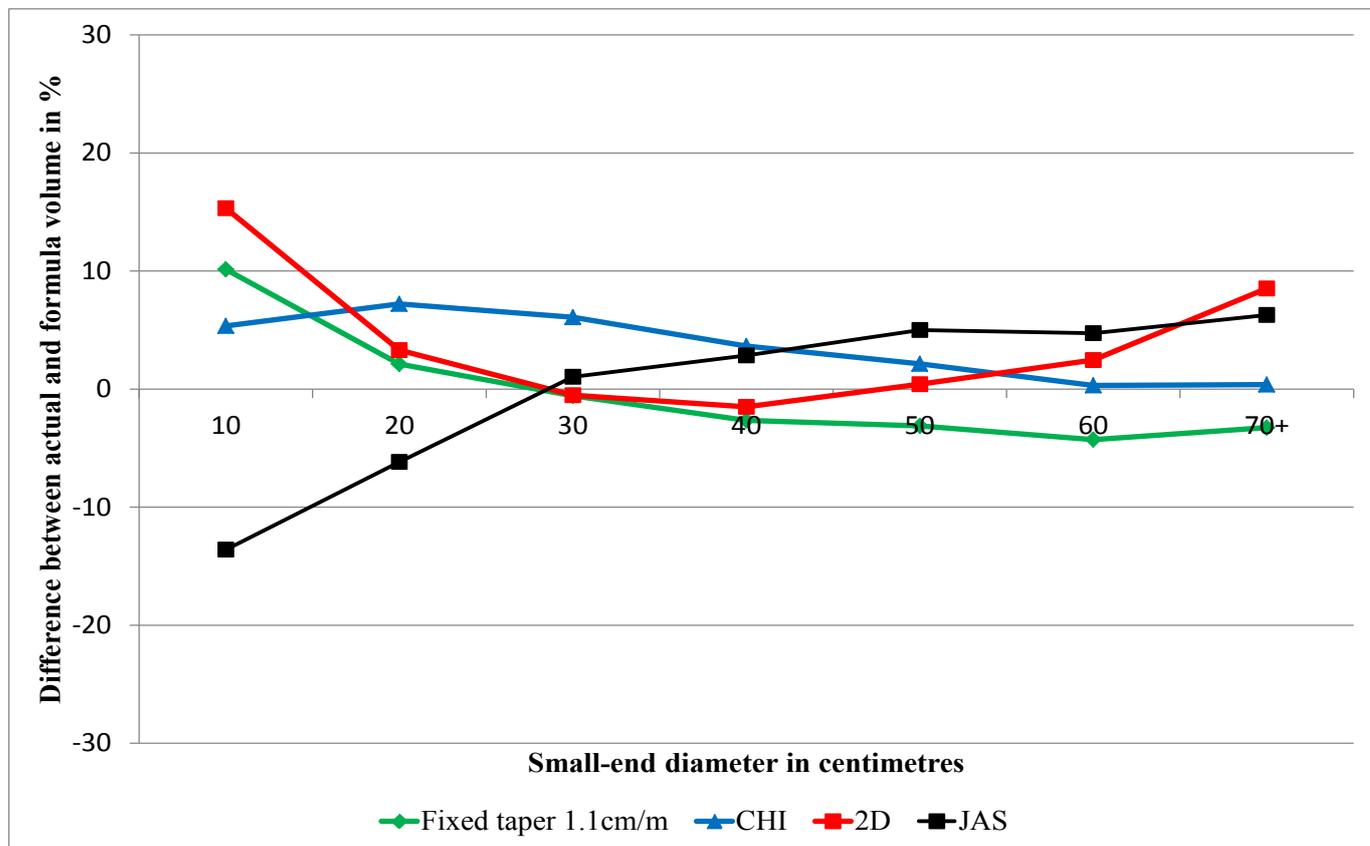


Figure 5. Volume errors on small-end diameter for external data set

Table 2. Overall bias of each method on data sets

Method	Data set		
	Basic data	Radiata pine (1946/80)	Douglas fir (1946/80)
	Bias	Bias	Bias
2D	-0.1%	+1.1%	+4.3%
2D fixed taper	+0.9%	-0.3%	+3.3%
JAS	+1.9%	-0.2%	+1.7%
CHI	+5.1%	+5.3	+5.2%
Logs	9163	35281	5733

The 2D formula shows a tendency to overestimate the extremes in diameter, whereas the other methods show more consistent error trends in volume over the small-end diameter range.

Note that bias is difference between methods and sectional volume for historical radiata pine and Douglas fir data sets. Bias on basic data is difference between methods and 3D volume.

In Table 2 the Chinese system shows a consistent bias of about 5% over all data sets. The 2D formula is unbiased on the basic data, but shows biases of +1.1% and +4.3% for the historical radiata pine and Douglas fir data sets respectively.

## Discussion and Conclusions

There is only a weak correlation between taper, small-end diameter and length in the data set and therefore the taper equation in this paper does not fully explain changes in taper. However, the inclusion of the taper formula as a taper term in the 3D formula to provide a new 2D formula provides a volume estimate over a normal range of export logs destined for sawn recovery, veneer and wood chip that does not exhibit the bias evident in JAS and the Chinese systems. The formula is not site specific and an average taper (based on small-end diameter and length) covers all sites. There is concern that the volume of logs from the higher altitude South Island sites with high taper will be underestimated. The change in volume estimate from 1.3cm/m taper to 1.4cm/m for the average log of 6.4 m in length and 32 cm in diameter is -1%, which is half the change in JAS volume for the same increase in taper.

Scaling rules for JAS are based on the shortest axis of the small-end diameter. This means that log makers need to allow about 2 additional centimetres so that the shortest axis and a scaled diameter rounded down to the even centimetre is not too small. Use of the 2D formula would remove the guesswork and the likelihood of producing logs below target diameter.

Many service costs are currently paid on JAS cubic metres. Some price adjustments will be needed to address increased volume and therefore increased costs for the smaller diameter logs. However, some

of the increased volume for the smaller diameter logs will be offset by eliminating some of the JAS volume overestimates for logs in some of the larger diameter classes. Logs are commonly purchased from the forest owner on the basis of JAS cubic metres, or JAS cubic metre per tonne values. For small diameter logs, JAS underestimates volume and therefore the forest owner is disadvantaged. Using the new formula, the forest owner will sell on a fairer volume estimate but the buyer would be disadvantaged with the increase in volume. Export log prices will need to be adjusted for the correction in volume.

At present two sets of conversions are required for yield reconciliations purposes. Log load weights are converted to volume under bark, and JAS volumes are converted to weight or cubic metres. The recommended export formula would remove the need for export volume to cubic metre conversion.

Basic data for the developed taper term were from managed stands of radiata pine, where the average taper is 1.3cm per metre of log length. The historical external data sets were comprised of logs from unthinned stands of trees in older age classes than are customary today. Given current silvicultural practices and philosophy it is unlikely that average log taper would decrease, or increase much from the current 1.3cm/m. Consequently it can be expected that the current taper term will continue to be appropriate for future log populations.

## Application and recommendations

Scaled JAS diameters used for volume calculation are based on the shortest axis inside the bark at the small-end of the log (Ellis, 1994). Diameters (for logs greater than 13cm in diameter) are rounded down to the nearest even centimetre class. Rounding down is easier for the scaler to apply than rounding diameters up and down to the mid-point of a diameter class, because the mid-point is always a smaller division on the ruler. However, as the average diameter for every class is one centimetre higher than that recorded, one centimetre is lost. For example, a recorded 26 cm covers all diameters between 26.0 cm and 27.99 cm, but the actual mean of the class is 27.0 cm. If the proposed formula is adopted it is recommended that the "rounded down rule" is retained, in which case one centimetre would need to be added to each diameter for volume calculation purposes.

Much effort is expended in measuring the diameters of small logs which experience shows tend to be circular in cross-section. Therefore, it would be practical for the scaler to measure one average diameter (instead of two at right angles) for logs less than 18 centimetres in small-end diameter.

### Acknowledgements

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## Appendix One

**The Volume Formulae can be found on the next page.**

Appendix 1. Volume formulae

**PROPOSED 2D FORMULA**

$$V = \exp(1.944157 \cdot \ln(L) + 0.029931 \cdot D - 0.038675 + 0.884711 \cdot \ln(2.112 + 0.00071 \cdot D^2 - 0.0405 \cdot D - 0.047 \cdot L)) + 0.07854 \cdot D^2 \cdot L$$

Formula 2

where V is the log volume (dm<sup>3</sup>)  
 ln is the natural logarithm  
 exp is the antilog of natural logarithm  
 L is the log length (m)  
 D is the small-end diameter (cm)

**JAS FORMULAE**

**1. For logs less than 6 m long**

$$V = D^2 \cdot L / 10000$$

Formula 3

where D is the shortest diameter in centimetres  
 L = length in metres  
 V = volume in cubic metres

**2. For logs equal to or greater than 6 m long**

$$V = (D + (L - 4) / 2)^2 \cdot (L / 10000)$$

Formula 4

where D is the shortest diameter in centimetres  
 L is the length in metres  
 L` is the length in metres rounded down to whole metre.  
 V is the volume in cubic metres.  
 Note that (L` - 4) / 2 is the factor for taper.

**CHINESE NATIONAL STANDARD FORMULAE**

**1. For small-end diameter 4-12cm and lengths less than 10.2m:**

$$V = 0.7854 \cdot L \cdot (D + 0.45 \cdot L + 0.2)^2 / 10000$$

Formula 5

**2. For small-end diameter 14cm and greater and lengths between 2m and 10m:**

$$V = 0.7854 \cdot L \cdot [D + 0.5 \cdot L + 0.005 \cdot L^2 + 0.000125 \cdot L \cdot (14 - L)^2 \cdot (D - 10)]^2 / 10000$$

Formula 6

**3. For logs less than 2m and greater than 10.2m in length**

$$V = 0.8 \cdot L \cdot (D + 0.5 \cdot L)^2 / 10000$$

Formula 7

**4. For logs of 10.2m in length**

$$V = (V_{10.0} + V_{10.4}) / 2$$

Formula 8

where V is the log volume (m<sup>3</sup>)  
 V<sub>10.0</sub> and V<sub>10.4</sub> are volumes for log lengths of 10.0m and 10.4m logs.  
 L is the log length (m)  
 D is the small-end diameter (cm)